

Direct CP violation in $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$

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Abstract

We study the direct CP violation in $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ (with unpolarized $\rho^0(\omega)$) via the $\rho - \omega$ mixing mechanism which causes a large strong phase difference and consequently a large CP violating asymmetry when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. Since there are two $\rho(\omega)$ mesons in the intermediate state $\rho - \omega$ mixing contributes twice to the first order of isospin violation, leading to an even larger CP violating asymmetry (could be 30% – 50% larger) than in the case where only one $\rho(\omega)$ meson is involved. The CP violating asymmetry depends on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the hadronic matrix elements. The factorization approach is applied in the calculation of the hadronic matrix elements with the nonfactorizable effects being included effectively in an effective parameter, N_c . We give the constraint on the range of N_c from the latest experimental data for the branching ratios for $\bar{B}^0 \rightarrow \rho^0\rho^0$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$. We find that the CP violating asymmetry could be very large (even more than 90% for some values of N_c). It is shown that the sensitivity of the CP violating asymmetry to N_c is large compared with its smaller sensitivity to the CKM matrix elements. We also discuss the possibility to remove the mod (π) ambiguity in the determination of the CP violating phase angle α through the measurement of the CP violating asymmetry in the decay $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$.

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I. INTRODUCTION

Although CP violation has been a central concern in particle physics since it was first observed in the neutral kaon system more than four decades ago [1] the dynamical origin of CP violation still remains an open problem. CP violation in the framework of the Standard Model (SM) is supposed to arise from a weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix which is based on quark flavor mixing [2, 3]. Therefore, the study of CP violation is essential to the test of the CKM mechanism in the SM.

Besides the kaon system much more studies have been carried out on CP violation in the B meson system both theoretically and experimentally in the past few years. It was suggested theoretically that large CP violating asymmetries should be observed in the experiments for B mesons [4]. This important prediction has already been confirmed by the experiments of BaBar and Belle etc. through the measurements on CP violation in several decay channels of B mesons such as $B^0 \rightarrow J/\psi K_S^0$ and $B^0 \rightarrow K^+\pi^-$ [5]. From the summer of 2007, the Large Hadron Collider (LHC) at CERN will start to contribute to the exploration of CP violation in the B meson system in a more accurate way due to its much higher statistics. This will also provide an opportunity to discover new physics beyond the SM.

In the decay process we have the so-called direct CP violation which occurs through the interference of two amplitudes with different weak phases and strong phases. The weak phase difference is directly determined by the CKM matrix. On the contrary, the strong phase is usually due to complicated strong interaction and hence difficult to control. Since a large strong phase difference is required for a large CP asymmetry, one needs to appeal to some phenomenological mechanism to get such a large strong phase difference. The charge asymmetry violating mixing between ρ^0 and ω ($\rho - \omega$ mixing) has been applied for this purpose in the past few years. From a series of studies for CP violation in some decay channels of heavy hadrons including B , Λ_b and D , it has been found that $\rho - \omega$ mixing can provide a very large strong phase difference (usually 90 degrees) when the mass of the decay product of $\rho(\omega)$, $\pi^+\pi^-$, is in the vicinity of the ω resonance [6, 7, 8, 9, 10]. Furthermore, it has been shown that the measurement of the CP violating asymmetry for these decays can be used to remove the mod (π) ambiguity in the determination of the CP violating phase angle α .

In this paper, we will investigate the CP violating asymmetry for the decay $\bar{B}^0 \rightarrow$

$\rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. This process is unique since it has two $\rho(\omega)$ mesons in the intermediate state, each of them contributing $\rho - \omega$ mixing. One can expect that there should be a bigger CP violating asymmetry than in the case where $\rho - \omega$ mixing only contributes once. It will be shown from our explicit calculations that this is true indeed. The CP violating asymmetry in the case of double $\rho - \omega$ mixing could be 30 – 50% bigger than that in the case of single $\rho - \omega$ mixing, depending on the value of N_c and q^2/m_b^2 (see the meaning of N_c and q^2/m_b^2 below).

In our calculations of the CP violating asymmetry, hadronic matrix elements for both tree and penguin operators in the effective Hamiltonian are involved. These matrix elements are controlled by the effects of nonperturbative QCD which are difficult to handle. In order to extract the strong phase difference we will use the factorization approximation, in which one of the currents in the Hamiltonian is factorized out and generates a meson, assuming the vacuum intermediate state saturation. In this way, the decay amplitude becomes the product of two matrix elements. Such factorization scheme was first argued to be plausible in energetic decays like bottom-hadron decays [11][12], then was proved to be the leading order result in the framework of QCD factorization when the radiative QCD corrections of order $O(\alpha_s(m_b))$ (m_b is the b-quark mass) and the $O(1/m_b)$ corrections in the heavy quark effective theory are neglected [13]. Since the nonfactorizable contributions are ignored in the factorization scheme we introduce an effective parameter, N_c , in order to take into account nonfactorizable contributions effectively. In this way, the value of N_c is not the color number (3) any more, but should be determined by experimental data. In the present work, this will be done by comparing the theoretical results with the experimental data for the decay branching ratios for the processes $\bar{B}^0 \rightarrow \rho^0\rho^0$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$.

The remainder of this paper is organized as follows. In Sec. II we briefly present the effective Hamiltonian, the Wilson coefficients and the CKM matrix elements. In Sec. III we give the formalism for the CP violating asymmetry in $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ via $\rho - \omega$ mixing. Then we give the calculation details of the strong phase difference and the numerical results for the CP violating asymmetry. In Sec. IV, we calculate the branching ratios for $\bar{B}^0 \rightarrow \rho^0\rho^0$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$ and present the range of N_c allowed by the latest experimental data for these decays. In the last section, we give a summary and discussion.

II. THE EFFECTIVE HAMILTONIAN AND THE CKM MATRIX

In order to calculate the direct CP violating asymmetry one needs to use the following effective weak Hamiltonian based on the operator product expansion [14]:

$$H_{\Delta B=1} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} V_{ub} V_{uq}^* (c_1 O_1^u + c_2 O_2^u) - V_{tb} V_{tq}^* \sum_{i=3}^{10} c_i O_i \right] + H.c., \quad (1)$$

where c_i ($i=1, \dots, 10$) are the Wilson coefficients, V_{ub} , V_{uq} , V_{tb} and V_{tq} are the CKM matrix elements. The operators O_i have the following form:

$$\begin{aligned} O_1^u &= \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha, \\ O_2^u &= \bar{q} \gamma_\mu (1 - \gamma_5) u \bar{u} \gamma^\mu (1 - \gamma_5) b, \\ O_3 &= \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', \\ O_4 &= \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \\ O_5 &= \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', \\ O_6 &= \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\ O_7 &= \frac{3}{2} \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', \\ O_8 &= \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\ O_9 &= \frac{3}{2} \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', \\ O_{10} &= \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \end{aligned} \quad (2)$$

where α and β are color indices, and $q' = u, d$ or s quarks. In Eq. (2) O_1^u and O_2^u are tree operators, O_3 – O_6 are QCD penguin operators, and O_7 – O_{10} arise from electroweak penguin diagrams.

The Wilson coefficients are known to the next-to-leading logarithmic order [14, 15]. They are renormalization scheme dependent since the renormalization prescription involves an arbitrariness in the finite parts in the renormalization procedure. The physical quantities should be renormalization scheme independent. Since the radiative QCD corrections are not included in the factorization approach we work, the hadronic matrix elements do not

carry any information about the renormalization scheme dependence¹. Therefore, we choose to use the renormalization scheme independent Wilson coefficients which are defined in Refs. [15, 16, 17] so that the CP violating asymmetry we obtain is renormalization scheme independent. The renormalization scale μ is chosen as the energy scale in the decays of the B meson, $O(m_b)$. When $\mu = 5$ GeV, these renormalization scheme independent Wilson coefficients take the following values [16, 17]:

$$\begin{aligned}
c_1 &= -0.3125, & c_2 &= 1.1502, \\
c_3 &= 0.0174, & c_4 &= -0.0373, \\
c_5 &= 0.0104, & c_6 &= -0.0459, \\
c_7 &= -1.050 \times 10^{-5}, & c_8 &= 3.839 \times 10^{-4}, \\
c_9 &= -0.0101, & c_{10} &= 1.959 \times 10^{-3}.
\end{aligned} \tag{3}$$

The matrix elements of the operators O_i should be renormalized to the one-loop order. This results in the effective Wilson coefficients, c'_i , which satisfy the constraint

$$c_i(m_b)\langle O_i(m_b) \rangle = c'_i\langle O_i \rangle^{\text{tree}}, \tag{4}$$

where $\langle O_i \rangle^{\text{tree}}$ are the matrix elements at the tree level, which will be evaluated in the factorization approach. From Eq. (4), the relations between c'_i and c_i are [16, 17]

$$\begin{aligned}
c'_1 &= c_1, & c'_2 &= c_2, \\
c'_3 &= c_3 - P_s/3, & c'_4 &= c_4 + P_s, \\
c'_5 &= c_5 - P_s/3, & c'_6 &= c_6 + P_s, \\
c'_7 &= c_7 + P_e, & c'_8 &= c_8, \\
c'_9 &= c_9 + P_e, & c'_{10} &= c_{10},
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
P_s &= (\alpha_s/8\pi)c_2[10/9 + G(m_c, \mu, q^2)], \\
P_e &= (\alpha_{em}/9\pi)(3c_1 + c_2)[10/9 + G(m_c, \mu, q^2)],
\end{aligned}$$

¹ It has been shown that in the QCD factorization approach the renormalization scheme dependence of the Wilson coefficients and that of the hadronic matrix elements cancel [13].

with

$$G(m_c, \mu, q^2) = 4 \int_0^1 dx x(x-1) \ln \frac{m_c^2 - x(1-x)q^2}{\mu^2},$$

where m_c is the c -quark mass and q^2 is the typical momentum transfer of the gluon or photon in the penguin diagrams. $G(m_c, \mu, q^2)$ has the following explicit expression [18]:

$$\begin{aligned} \Re G &= \frac{2}{3} \left(\ln \frac{m_c^2}{\mu^2} - \frac{5}{3} - 4 \frac{m_c^2}{q^2} + \left(1 + 2 \frac{m_c^2}{q^2} \right) \sqrt{1 - 4 \frac{m_c^2}{q^2}} \ln \frac{1 + \sqrt{1 - 4 \frac{m_c^2}{q^2}}}{1 - \sqrt{1 - 4 \frac{m_c^2}{q^2}}} \right), \\ \Im G &= -\frac{2}{3} \pi \left(1 + 2 \frac{m_c^2}{q^2} \right) \sqrt{1 - 4 \frac{m_c^2}{q^2}}. \end{aligned} \quad (6)$$

The value of q^2 is chosen to be in the range $0.3 < q^2/m_b^2 < 0.5$ [6, 7]. From Eqs. (3) (5) (6) we can obtain numerical values of c'_i which are listed in Table I, where we have taken $\alpha_s(m_Z)=0.118$, $\alpha_{em}(m_b)=1/132.2$, $m_b=5$ GeV, and $m_c=1.35$ GeV.

TABLE I: Effective Wilson coefficients for the tree operators, electroweak and QCD penguin operators [17, 18]

c'_i	$q^2/m_b^2=0.3$	$q^2/m_b^2=0.5$
c'_1	-0.3125	-0.3125
c'_2	1.1502	1.1502
c'_3	$2.433 \times 10^{-2} + 1.543 \times 10^{-3}i$	$2.120 \times 10^{-2} + 5.174 \times 10^{-3}i$
c'_4	$-5.808 \times 10^{-2} - 4.628 \times 10^{-3}i$	$-4.869 \times 10^{-2} - 1.552 \times 10^{-2}i$
c'_5	$1.733 \times 10^{-2} + 1.543 \times 10^{-3}i$	$1.420 \times 10^{-2} + 5.174 \times 10^{-3}i$
c'_6	$-6.668 \times 10^{-2} - 4.628 \times 10^{-3}i$	$-5.729 \times 10^{-2} - 1.552 \times 10^{-2}i$
c'_7	$-1.435 \times 10^{-4} - 2.963 \times 10^{-5}i$	$-8.340 \times 10^{-5} - 9.938 \times 10^{-5}i$
c'_8	3.839×10^{-4}	3.839×10^{-4}
c'_9	$-1.023 \times 10^{-2} - 2.963 \times 10^{-5}i$	$-1.017 \times 10^{-2} - 9.938 \times 10^{-5}i$
c'_{10}	1.959×10^{-3}	1.959×10^{-3}

The CKM matrix, which should be determined from experiments, can be expressed in terms of the Wolfenstein parameters, A, λ, ρ and η [19]:

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (7)$$

where $O(\lambda^4)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [20]:

$$\begin{aligned} \lambda &= 0.2272 \pm 0.0010, \quad A = 0.818^{+0.007}_{-0.017}, \\ \bar{\rho} &= 0.221^{+0.064}_{-0.028}, \quad \bar{\eta} = 0.340^{+0.017}_{-0.045}, \end{aligned} \quad (8)$$

where

$$\bar{\rho} = \rho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \quad (9)$$

From Eqs. (8) (9) we have

$$0.198 < \rho < 0.293, \quad 0.302 < \eta < 0.366. \quad (10)$$

III. CP VIOLATION IN $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$

A. Formalism

Letting A (\bar{A}) be the amplitude for the decay $\bar{B}^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ ($B^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$) one has:

$$A = \langle \pi^+\pi^-\pi^+\pi^- | H^T | \bar{B}^0 \rangle + \langle \pi^+\pi^-\pi^+\pi^- | H^P | \bar{B}^0 \rangle, \quad (11)$$

$$\bar{A} = \langle \pi^+\pi^-\pi^+\pi^- | H^T | B^0 \rangle + \langle \pi^+\pi^-\pi^+\pi^- | H^P | B^0 \rangle, \quad (12)$$

with H^T and H^P being the Hamiltonian for the tree and penguin operators, respectively.

We can define the relative magnitude and phases between the tree and penguin operator contributions as follows:

$$A = \langle \pi^+\pi^-\pi^+\pi^- | H^T | \bar{B}^0 \rangle [1 + re^{i(\delta+\phi)}], \quad (13)$$

$$\bar{A} = \langle \pi^+\pi^-\pi^+\pi^- | H^T | B^0 \rangle [1 + re^{i(\delta-\phi)}], \quad (14)$$

where δ and ϕ are strong and weak relative phases, respectively. The phase ϕ can be expressed as a combination of the CKM matrix elements: $\phi = \arg[(V_{tb}V_{td}^*)/(V_{ub}V_{ud}^*)]$. As a

result, $\sin\phi$ is equal to $\sin\alpha$ with α being defined in the standard way [20]. The parameter r is the absolute value of the ratio of penguin and tree amplitudes:

$$r \equiv \left| \frac{\langle \pi^+ \pi^- \pi^+ \pi^- | H^P | \bar{B}^0 \rangle}{\langle \pi^+ \pi^- \pi^+ \pi^- | H^T | \bar{B}^0 \rangle} \right|. \quad (15)$$

The CP violating asymmetry, a , can be written as

$$a \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r \sin\delta \sin\phi}{1 + 2r \cos\delta \cos\phi + r^2}. \quad (16)$$

In order to obtain a large signal for direct CP violation, we need some mechanism to make $\sin\delta$ large. It has been found that $\rho - \omega$ mixing has the dual advantages that it leads to a large strong phase difference and is well known [7, 8, 9, 10]. With this mechanism, to the first order of isospin violation, we have the following results when the invariant masses of $\pi^+ \pi^-$ pairs are near the ω resonance mass:

$$\langle \pi^+ \pi^- \pi^+ \pi^- | H^T | \bar{B}^0 \rangle = \frac{2g_\rho^2}{s_\rho^2 s_\omega} \tilde{\Pi}_{\rho\omega} t_{\rho\omega} + \frac{g_\rho^2}{s_\rho^2} t_{\rho\rho}, \quad (17)$$

$$\langle \pi^+ \pi^- \pi^+ \pi^- | H^P | \bar{B}^0 \rangle = \frac{2g_\rho^2}{s_\rho^2 s_\omega} \tilde{\Pi}_{\rho\omega} p_{\rho\omega} + \frac{g_\rho^2}{s_\rho^2} p_{\rho\rho}. \quad (18)$$

Here $t_{\rho\rho}(p_{\rho\rho})$ and $t_{\rho\omega}(p_{\rho\omega})$ are the tree (penguin) amplitudes for $\bar{B} \rightarrow \rho^0 \rho^0$ and $\bar{B}^0 \rightarrow \rho^0 \omega$, respectively, g_ρ is the coupling for $\rho^0 \rightarrow \pi^+ \pi^-$, $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho - \omega$ mixing amplitude which also effectively includes the direct coupling $\omega \rightarrow \pi^+ \pi^-$, and $s_V(V=\rho \text{ or } \omega)$ is the inverse propagator of the vector meson V ,

$$s_V = s - m_V^2 + im_V \Gamma_V, \quad (19)$$

with \sqrt{s} being the invariant masses of the $\pi^+ \pi^-$ pairs (we let the invariant masses of the two $\pi^+ \pi^-$ pairs be the same). Eqs. (17) (18) have different forms from the case where only single $\rho - \omega$ mixing is involved [8, 9, 10]: there is a factor of 2 in front of the effective $\rho - \omega$ mixing amplitude, $\tilde{\Pi}_{\rho\omega}$, since $\rho - \omega$ mixing contributes twice to the first order of isospin violation. Furthermore, we have g_ρ^2 and s_ρ^2 instead of g_ρ and s_ρ as before due to two $\rho \rightarrow \pi\pi$ couplings and two ρ propagators (note that s_ω^2 term is of the second order of isospin violation and hence is ignored).

As mentioned before, the direct coupling $\omega \rightarrow \pi^+ \pi^-$ has been effectively absorbed into $\tilde{\Pi}_{\rho\omega}$ [21]. This leads to the explicit s dependence of $\tilde{\Pi}_{\rho\omega}$. In practice, however, the s

dependence of $\tilde{\Pi}_{\rho\omega}$ is negligible. Making the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$, the $\rho - \omega$ mixing parameters were determined in the fit of Gardner and O'Connell [22]:

$$\begin{aligned}\Re\tilde{\Pi}_{\rho\omega}(m_\omega^2) &= -3500 \pm 300 \text{MeV}^2, \\ \Im\tilde{\Pi}_{\rho\omega}(m_\omega^2) &= -300 \pm 300 \text{MeV}^2, \\ \tilde{\Pi}'_{\rho\omega}(m_\omega^2) &= 0.03 \pm 0.04.\end{aligned}\tag{20}$$

From Eqs. (11)(13)(17)(18) one has

$$re^{i\delta}e^{i\phi} = \frac{2\tilde{\Pi}_{\rho\omega}p_{\rho\omega} + s_\omega p_{\rho\rho}}{2\tilde{\Pi}_{\rho\omega}t_{\rho\omega} + s_\omega t_{\rho\rho}},\tag{21}$$

where the factor of 2 in front of $\tilde{\Pi}_{\rho\omega}$ arises from the involvement of double $\rho - \omega$ mixing. Defining

$$\frac{p_{\rho\omega}}{t_{\rho\rho}} \equiv r'e^{i(\delta_q + \phi)}, \quad \frac{t_{\rho\omega}}{t_{\rho\rho}} \equiv \alpha e^{i\delta_\alpha}, \quad \frac{p_{\rho\rho}}{p_{\rho\omega}} \equiv \beta e^{i\delta_\beta},\tag{22}$$

where δ_α , δ_β and δ_q are strong phases, one finds the following expression from Eqs. (21)(22):

$$re^{i\delta} = r'e^{i\delta_q} \frac{2\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta} s_\omega}{2\tilde{\Pi}_{\rho\omega} \alpha e^{i\delta_\alpha} + s_\omega}.\tag{23}$$

In order to obtain the CP violating asymmetry in Eq. (16), $\sin\phi$ and $\cos\phi$ are needed, where ϕ is determined by the CKM matrix elements. In the Wolfenstein parametrization [19], one has

$$\sin\phi = \frac{\eta}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}},\tag{24}$$

$$\cos\phi = \frac{\rho(1-\rho) - \eta^2}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}.\tag{25}$$

B. Computational details

With the Hamiltonian given in Eq. (1) we can evaluate the matrix elements for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega)$. In the factorization approximation, $\rho^0(\omega)$ is generated by one current which has the appropriate quantum numbers in the Hamiltonian. For this decay process, the amplitude can be written as the product of two matrix elements after factorization, i.e. (omitting Dirac matrices and color labels): $\langle\rho^0(\omega)|(\bar{q}q)|0\rangle\langle\rho^0(\omega)|(\bar{d}b)|\bar{B}^0\rangle$ ($q = u, d$), where $(\bar{q}q)$ and $(\bar{d}b)$ denote the $V - A$ currents, $\bar{q}\gamma_\mu(1 - \gamma_5)q$ and $\bar{d}\gamma_\mu(1 - \gamma_5)b$, respectively. Since ρ^0 and

ω are vector mesons the amplitude for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega)$ may be polarized or unpolarized. Here we investigate the later case. Defining

$$\langle \rho^0 | (\bar{u}u) | 0 \rangle \langle \rho^0 | (\bar{d}b) | \bar{B}^0 \rangle \equiv T, \quad (26)$$

one has

$$\begin{aligned} T &= -\langle \rho^0 | (\bar{d}d) | 0 \rangle \langle \rho^0 | (\bar{d}b) | \bar{B}^0 \rangle \\ &= -\langle \rho^0 | (\bar{u}u) | 0 \rangle \langle \omega | (\bar{d}b) | \bar{B}^0 \rangle \\ &= \langle \rho^0 | (\bar{d}d) | 0 \rangle \langle \omega | (\bar{d}b) | \bar{B}^0 \rangle \\ &= \langle \omega | (\bar{u}u) | 0 \rangle \langle \rho^0 | (\bar{d}b) | \bar{B}^0 \rangle. \end{aligned} \quad (27)$$

After factorization, the contribution to $t_{\rho\rho}$ from the tree level operator O_1^u is

$$\langle \rho^0 \rho^0 | O_1^u | \bar{B}^0 \rangle = 2 \langle \rho^0 | (\bar{u}u) | 0 \rangle \langle \rho^0 | (\bar{d}b) | \bar{B}^0 \rangle = 2T. \quad (28)$$

Using the Fierz transformation the contribution of O_2^u is $(1/N_c)T$. Hence we have

$$t_{\rho\rho} = 2 \left(c_1' + \frac{1}{N_c} c_2' \right) T. \quad (29)$$

It should be noted that in Eq. (29) we have neglected the color-octet contribution which is nonfactorizable and difficult to calculate. Therefore, N_c should be treated as an effective parameter and may deviate from the naive value 3 [8, 9, 10]. In the same way we find that $t_{\rho\omega} = 0$. This lead to

$$\alpha e^{i\delta_\alpha} = 0, \quad (30)$$

from Eq. (22).

In a similar way, we can evaluate the penguin operator contributions $p_{\rho\rho}$ and $p_{\rho\omega}$ with the aid of the Fierz identities. From Eq. (22) we have

$$\begin{aligned} \beta e^{i\delta_\beta} &= \frac{-2 \left(c_4' + \frac{1}{N_c} c_3' \right) + 3 \left(c_7' + \frac{1}{N_c} c_8' \right) + 3 \left(c_9' + \frac{1}{N_c} c_{10}' \right) + \left(c_{10}' + \frac{1}{N_c} c_9' \right)}{2 \left(c_3' + \frac{1}{N_c} c_4' \right) + 2 \left(c_4' + \frac{1}{N_c} c_3' \right) + 2 \left(c_5' + \frac{1}{N_c} c_6' \right) - \left(c_7' + \frac{1}{N_c} c_8' \right) - \left(c_9' + \frac{1}{N_c} c_{10}' \right) - \left(c_{10}' + \frac{1}{N_c} c_9' \right)}, \\ r' e^{i\delta_q} &= \frac{-2 \left(c_3' + \frac{1}{N_c} c_4' \right) - 2 \left(c_4' + \frac{1}{N_c} c_3' \right) - 2 \left(c_5' + \frac{1}{N_c} c_6' \right) + \left(c_7' + \frac{1}{N_c} c_8' \right) + \left(c_9' + \frac{1}{N_c} c_{10}' \right) + \left(c_{10}' + \frac{1}{N_c} c_9' \right)}{2 \left(c_1' + \frac{1}{N_c} c_2' \right)} \end{aligned} \quad (31)$$

$$\times \left| \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right|, \quad (32)$$

where

$$\left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right| = \frac{\sqrt{(1-\rho)^2 + \eta^2}}{(1-\lambda^2/2)\sqrt{\rho^2 + \eta^2}}. \quad (33)$$

C. Numerical results

We have several parameters in the numerical calculations: q^2 , N_c , and the CKM matrix elements. Since N_c includes nonfactorizable effects, which cannot be evaluated accurately at present, we choose to treat it as a parameter and determine its range from the experimental data. Then, we can extract an allowed range for N_c from a comparison of the theoretical results and the experimental data. By doing this, we get the range of N_c as $2.74(2.81) < N_c < 4.77(4.92)$ for $q^2/m_b^2 = 0.3(0.5)$. This will be discussed in detail in Sec. IV. The most uncertainties due to the CKM matrix elements come from ρ and η since λ is well determined (see Eq. (8)) and since the CP violating asymmetry is independent of the Wolfenstein parameter A . Therefore, in our numerical calculations, we take the central value for λ and only let (ρ, η) vary between the limiting values (ρ_{min}, η_{min}) and (ρ_{max}, η_{max}) . In fact, explicit numerical results show that the CP violating asymmetry is very insensitive to λ .

In the numerical calculations, it is found that for a fixed N_c there is a maximum value, a_{max} , for the CP violating parameter, a , when the invariant masses of the $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. This is shown explicitly in Fig. 1. For $q^2/m_b^2 = 0.3(0.5)$ and $N_c = 2.74(2.81)$, the maximum CP violating asymmetry varies from around -91.1% (-70.1%) to around -96.1% (-77.8%) as (ρ, η) change from (ρ_{max}, η_{max}) to (ρ_{min}, η_{min}) ; For $q^2/m_b^2 = 0.3(0.5)$ and $N_c = 4.77(4.92)$, the maximum CP violating asymmetry varies from around 55.8% (28.9%) to around 53.2% (22.9%) when (ρ, η) change from (ρ_{max}, η_{max}) to (ρ_{min}, η_{min}) .

Our results show that the $\rho - \omega$ mixing mechanism produces a large $\sin\delta$ in the allowed range of N_c , which is necessary for a large CP violating asymmetry. The involvement of double $\rho - \omega$ mixing in $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ gives rise to a factor of 2 in Eq. (21) in front of $\tilde{\Pi}_{\rho\omega}$. This makes the CP asymmetry even larger than in the case where the $\rho - \omega$ mixing contributes only once. Fig. 2 shows explicitly the comparison between these two cases for $N_c = 2.74(2.81)$ and $q^2/m_b^2 = 0.3(0.5)$. The maximum asymmetry with the involvement of single $\rho - \omega$ mixing, for $q^2/m_b^2 = 0.3(0.5)$ and $N_c = 2.74(2.81)$, is around -55.2% (-20.0%) for the set (ρ_{max}, η_{max}) and -63.2% (-23.8%) for the set (ρ_{min}, η_{min}) . For

$q^2/m_b^2 = 0.3(0.5)$ and $N_c = 4.77(4.92)$, we find that a_{max} is around 26.5% (-1.49%) for the set (ρ_{max}, η_{max}) and 25.1% (-1.50%) for the set (ρ_{min}, η_{min}) in the case of single $\rho - \omega$ mixing.

The reason that double $\rho - \omega$ mixing leads to a larger CP violating asymmetry than in the case of single $\rho - \omega$ mixing is that $\sin\delta$ becomes bigger in the case of double $\rho - \omega$ mixing than in the case of single $\rho - \omega$ mixing. This can be seen explicitly from Fig. 3. The involvement of double $\rho - \omega$ mixing may also change the value of r as can be seen from Fig. 4. However, as found from our detailed analysis for the influence of r on the CP violating asymmetry, the effect of the change of r on a is small compared with the change of $\sin\delta$ due to the involvement of double $\rho - \omega$ mixing.

It is noted that when N_c is around 2.81 and 4.92 in the case $q^2/m_b^2 = 0.5$, we could also have large CP asymmetries when \sqrt{s} is far away from the ω resonance for all the allowed values of the CKM matrix elements (see Figs. 1(b) and 5(b)). In these cases, the effective $\rho - \omega$ mixing contributes little and the large CP asymmetry is caused by the effective Wilson coefficients, which can also give a large strong phase, δ , since they are complex numbers.

In most direct CP violating decays such as $\bar{B}^0 \rightarrow \rho^0(\omega)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ [9, 10] and some other processes, the involvement of $\rho - \omega$ mixing leads to the result that the strong phase, δ , passes through 90° ($\sin\delta=1$) at the ω resonance. However, in the decay we are discussing this does not happen in the allowed range of N_c . Instead, the absolute value of $\sin\delta$ just gets close to 1, but does not equal 1 (see Fig. 3), even though it is enough to give large CP asymmetry, especially when double $\rho - \omega$ mixing is involved.

Figs. 5 and 6 show the dependence of the CP violating asymmetry and $\sin\delta$, respectively, on both N_c and \sqrt{s} . One can see that the CP asymmetry strongly depends on N_c . Take Fig. 5(a) as an example (for $q^2/m_b^2 = 0.3$ and maximum ρ and η): when $N_c < 3.68$, one gets minus asymmetry around the ω resonance, whereas when $N_c > 3.68$ the CP violating asymmetry becomes positive.

It can be seen from Fig. 5 that when N_c takes the critical value, $-c'_2/c'_1 \simeq 3.68$, the CP violating asymmetry becomes zero. This is because $t_{\rho\rho} = 0$ at this point (as can be seen from Eq. (29) easily) and hence the penguin operator contributions dominate. Furthermore, the sign of $\sin\delta$ and hence the sign of the CP violating asymmetry change at this point. It would be interesting to see whether or not N_c can take this value in the future when more accurate experimental data are available. From most previous studies, it seems that N_c is usually less than this critical value [7, 8, 9, 10, 23]. If this is true for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega)$, then

the sign of $\sin\delta$ would remain unchanged. Then, one could remove the mod (π) ambiguity in the determination of the CP violating phase angle α (through $\sin 2\alpha$) by measuring the CP violating asymmetry in $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$.

IV. BRANCHING RATIOS FOR $\bar{B}^0 \rightarrow \rho^0\rho^0$ AND $\bar{B}^0 \rightarrow \rho^+\rho^-$

A. Formalism

If the decay amplitude for $B \rightarrow V_1V_2$ (V_1, V_2 denote vector mesons) has the form $A(B \rightarrow V_1V_2) = \alpha X^{(BV_1, V_2)}$, where $X^{(BV_1, V_2)}$ denotes the factorizable amplitude with the form $\langle V_2 | (\bar{q}_2 q_3) | 0 \rangle \langle V_1 | (\bar{q}_1 b) | B \rangle$, then the decay rate is given by [23]

$$\Gamma(B \rightarrow V_1V_2) = \frac{p_c}{8\pi m_B^2} |\alpha(m_B + m_1)m_2 f_{V_2} A_1^{BV_1}(m_2)|^2 H, \quad (34)$$

where α is related to the CKM matrix elements and Wilson coefficients, f_{V_2} is the decay constant of V_2 , p_c is the c.m. momentum of the decay particles, m_B and $m_1(m_2)$ are the masses of the B meson and the vector meson $V_1(V_2)$, respectively, and

$$H = (a - bx)^2 + 2(1 - c^2 y^2), \quad (35)$$

where

$$\begin{aligned} a &= \frac{m_B^2 - m_1^2 - m_2^2}{2m_1 m_2}, \quad b = \frac{2m_B^2 p_c^2}{m_1 m_2 (m_B + m_1)^2}, \quad c = \frac{2m_B p_c}{(m_B + m_1)^2}, \\ x &= \frac{A_2^{BV_1}(m_2^2)}{A_1^{BV_1}(m_2^2)}, \quad y = \frac{V^{BV_1}(m_2^2)}{A_1^{BV_1}(m_2^2)}, \\ p_c &= \frac{\sqrt{[m_B^2 - (m_1 + m_2)^2][m_B^2 - (m_1 - m_2)^2]}}{2m_B}. \end{aligned} \quad (36)$$

$A_1^{BV_1}$, $A_2^{BV_1}$ and V^{BV_1} in Eqs. (35) and (36) are the form factors associated with $B \rightarrow V_1$ transition.

The decay amplitudes for $\bar{B}^0 \rightarrow \rho^0\rho^0$, $\bar{B}^0 \rightarrow \rho^0\omega$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$ are

$$A(\bar{B}^0 \rightarrow \rho^0\rho^0) = \alpha_1 X^{(B\rho^0, \rho^0)}, \quad (37)$$

$$A(\bar{B}^0 \rightarrow \rho^0\omega) = \alpha_2 X^{(B\rho^0, \omega)}, \quad (38)$$

and

$$A(\bar{B}^0 \rightarrow \rho^+\rho^-) = \alpha_3 X^{(B\rho^+, \rho^-)}, \quad (39)$$

where

$$\alpha_1 = \frac{G_F}{\sqrt{2}}[2a_1V_{ub}V_{ud}^* - (-2a_4 + 3a_7 + 3a_9 + a_{10})V_{tb}V_{td}^*], \quad (40)$$

$$\alpha_2 = -\frac{G_F}{\sqrt{2}}(2a_3 + 2a_4 + 2a_5 - a_7 - a_9 - a_{10})V_{tb}V_{td}^*, \quad (41)$$

$$\alpha_3 = -\frac{G_F}{\sqrt{2}}[a_2V_{ub}V_{ud}^* - (a_4 + a_{10})V_{tb}V_{td}^*], \quad (42)$$

with a_i ($i = 1, 2, \dots, 10$) being defined as:

$$\begin{aligned} a_{2j} &= c'_{2j} + \frac{c'_{2j-1}}{N_c}, \\ a_{2j-1} &= c'_{2j-1} + \frac{c'_{2j}}{N_c}, \quad \text{for } j = 1, 2, \dots, 5. \end{aligned} \quad (43)$$

When we calculate the branching ratios we should take into account the $\rho - \omega$ mixing contribution for consistency since we are working to the first order of isospin violation. Then, we obtain the branching ratio for $\bar{B}^0 \rightarrow \rho^0 \rho^0$:

$$BR(\bar{B}^0 \rightarrow \rho^0 \rho^0) = \frac{p_c}{8\pi m_B^2 \Gamma_{B^0}} \left| \left(\alpha_1 + \alpha_2 \frac{2\tilde{\Pi}_{\rho\omega}}{(s_\rho - m_\omega^2) + im_\omega \Gamma_\omega} \right) (m_B + m_{\rho^0}) m_{\rho^0} f_{\rho^0} A_1(m_{\rho^0}^2) \right|^2 H. \quad (44)$$

For $\bar{B}^0 \rightarrow \rho^+ \rho^-$, we have

$$BR(\bar{B}^0 \rightarrow \rho^+ \rho^-) = \frac{p_c}{8\pi m_B^2 \Gamma_{B^0}} |(\alpha_3(m_B + m_{\rho^+}) m_{\rho^-} f_{\rho^-} A_1(m_{\rho^+}^2))|^2 H. \quad (45)$$

B. Form factor models

The form factors $A_1(k^2)$, $A_2(k^2)$ and $V(k^2)$ depend on the inner structure of hadrons and consequently depend on the phenomenological models for hadronic wave functions. We adopt the following form factor models:

Model 1(2) [24, 25]:

$$V(k^2) = \frac{V(0)}{1 - k^2/(m_{1-}^2)}, A_1(k^2) = \frac{A_1(0)}{1 - k^2/(m_{1+}^2)}, A_2(k^2) = \frac{A_2(0)}{1 - k^2/(m_{1+}^2)}, \quad (46)$$

where $V(0) = 0.33(0.395)$, $A_1(0) = A_2(0) = 0.28(0.345)$, $m_{1-} = 5.32\text{GeV}$, and $m_{1+} = 5.71\text{GeV}$.

Model 3(4) [23, 24, 25]:

$$V(k^2) = \frac{V(0)}{[1 - k^2/(m_{1-}^2)]^2}, A_1(k^2) = \frac{A_1(0)}{1 - k^2/(m_{1+}^2)}, A_2(k^2) = \frac{A_2(0)}{[1 - k^2/(m_{1+}^2)]^2}, \quad (47)$$

where the form factors have double pole dependence and the parameters take the same values as in Models 1 and 2.

Model 5 [26]:

for $V(k^2)$:

$$V(k^2) = \frac{V(0)}{(1 - k^2/m_V)[1 - \sigma_1 k^2/m_V^2 + \sigma_2 k^4/m_V^4]}, \quad (48)$$

for $A_i(k^2)$ ($i=1, 2$):

$$A_i(k^2) = \frac{A_i(0)}{1 - \sigma_1 k^2/m_V^2 + \sigma_2 k^4/m_V^4}, \quad (49)$$

where $m_V = m_{B^*} = 5.32\text{GeV}$; $V(0) = 0.31$, $\sigma_1 = 0.59$ and $\sigma_2 = 0$ for $V(k^2)$; $A_1(0) = 0.26$, $\sigma_1 = 0.73$ and $\sigma_2 = 0.10$ for $A_1(k^2)$; and $A_2(0) = 0.24$, $\sigma_1 = 1.40$ and $\sigma_2 = 0.50$ for $A_2(k^2)$.

Model 6 [27, 28]:

the form factors $A_1(k^2)$, $A_2(k^2)$ and $V(k^2)$ have the same form:

$$f(k^2) = \frac{f(0)}{1 - a_F k^2/m_B^2 + b_F k^4/m_B^4}, \quad (50)$$

where f could be A_1 , A_2 , or V . The parameters $f(0)$, a_F and b_F for various form factors are: for A_1 , $A_1(0) = 0.261$, $a_F = 0.29$, $b_F = -0.415$; for A_2 , $A_2(0) = 0.223$, $a_F = 0.93$, $b_F = -0.092$; and for V , $V(0) = 0.338$, $a_F = 1.37$, $b_F = 0.315$.

C. Numerical results

As mentioned before, N_c includes the nonfactorizable effects effectively, which cannot be handled well at present. Therefore, we treat N_c as a parameter to be determined by experimental data. Usually N_c is assumed to be universal for all decay channels in the factorization approach. However, it certainly could be different for different channels. Therefore, we choose to determine the range of N_c for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega)$ from the experimental data for the branching ratios for the decays $\bar{B}^0 \rightarrow \rho^0\rho^0$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$ (we expect the nonfactorizable contributions and hence the values of N_c in these two channels are the same if isospin violation is ignored). In order to find the range allowed for N_c we use the latest experimental data for the branching ratios for the two decay channels, $\bar{B}^0 \rightarrow \rho^0\rho^0$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$ [20]:

$$\begin{aligned} \text{BR}(\bar{B}^0 \rightarrow \rho^0\rho^0) &< 1.1 \times 10^{-6}, \\ \text{BR}(\bar{B}^0 \rightarrow \rho^+\rho^-) &= (2.5 \pm 0.4) \times 10^{-5}. \end{aligned} \quad (51)$$

We calculate the branching ratios for $\bar{B}^0 \rightarrow \rho^0 \rho^0$ and $\bar{B}^0 \rightarrow \rho^+ \rho^-$ with the formulae given in Eqs. (34) – (45) in all the models for the weak form factors associated with $\bar{B}^0 \rightarrow \rho^0$ and $\bar{B}^0 \rightarrow \rho^+(\rho^-)$, which are mentioned in the previous subsection. In addition to the dependence on N_c , these two branching ratios also depend on the CKM matrix elements which are parameterized by λ , A , ρ and η , with the experimental values of them being given in Eqs. (8) (10). Since each of these parameters has some uncertainty, we let each of them vary in its allowed range when we calculate the branching ratios. Then, for each set of the values for the parameters λ , A , ρ and η , we obtain a range of N_c which is allowed by the experimental data for the branching ratios for both $\bar{B}^0 \rightarrow \rho^0 \rho^0$ and $\bar{B}^0 \rightarrow \rho^+ \rho^-$. This is shown in Figs. 7 and 8 for a special set of CKM matrix parameters when $q^2/m_b^2 = 0.3$. Repeating this process for various sets of the values for λ , A , ρ and η and taking the union of the ranges of N_c for all these sets, we find a range for N_c which covers the whole range for these CKM matrix parameters. We repeat this process for all the form factor models mentioned in Eqs. (46) – (50) and obtain the range of N_c for each model as shown in Table II. Taking the union of all the ranges for these models we finally find the maximum possible range for N_c : $2.74 < N_c < 4.77$ and $2.81 < N_c < 4.92$ for $q^2/m_b^2 = 0.3$ and $q^2/m_b^2 = 0.5$, respectively.

TABLE II: The range of N_c for all the models and the maximum range of N_c .

	$q^2/m_b^2=0.3$	$q^2/m_b^2=0.5$
Model 1	(2.74, 4.74)	(2.81, 4.90)
Model 2	(2.78, 4.47)	(2.84, 4.64)
Model 3	(2.75, 4.77)	(2.81, 4.92)
Model 4	(2.76, 4.54)	(2.82, 4.72)
Model 5	(2.74, 4.69)	(2.81, 4.84)
Model 6	(2.77, 4.50)	(2.84, 4.68)
maximum range	(2.74, 4.77)	(2.81, 4.92)

V. SUMMARY AND DISCUSSION

We have calculated the CP violating asymmetry in the process $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ including the effect of $\rho - \omega$ mixing. The advantage of $\rho - \omega$ mixing is that it

makes the strong phase difference, δ , between the hadronic matrix elements of the tree and penguin operators very large at the ω resonance for a fixed N_c . We have found that $\sin\delta$ becomes large and reaches the maximum point at the ω resonance. Consequently, the CP violating asymmetry reaches the maximum value when the invariant masses of the $\pi^+\pi^-$ pairs in the decay product are in the vicinity of the ω resonance. Furthermore, since there are two $\rho(\omega)$ mesons in the intermediate state, $\rho-\omega$ mixing contributes twice when we work to the first order of isospin violation. This leads to an even larger CP violating asymmetry than in the case where only single $\rho-\omega$ mixing is involved. This is unique for the process $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. As a result, the largest CP violating asymmetry could be more than 90% for some values of N_c . This could be observed in the future experiments at LHC. Now, we roughly estimate the possibility to observe this CP violating asymmetry. If the branching ratio for $\bar{B}^0 \rightarrow \rho^0\rho^0$ is of order 10^{-6} , then the number of $B^0\bar{B}^0$ pairs needed for observing the CP violating asymmetry (90%) is roughly $\frac{1}{BR(B^0 \rightarrow \rho^0\rho^0)} \frac{1}{a^2} \sim 10^6$ for 1σ signature and 10^7 for 3σ signature [29]. It has been pointed out that at LHC, the number of $B^0\bar{B}^0$ pairs could be around 4×10^7 (for ATLAS and CMS) and 4×10^5 (for LHCb) per year [30]. Therefore, it is possible to observe the CP violation for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ via the double $\rho-\omega$ mixing mechanism at LHC.

In the calculation, we need the Wilson coefficients for the tree and penguin operators at the scale m_b . We work with the renormalization scheme independent Wilson coefficients. We have found that apart from the $\rho-\omega$ mixing mechanism, the Wilson coefficients themselves could also give observable CP violating asymmetry in some cases. The errors in the CKM matrix elements lead to some uncertainties in the CP violating asymmetry. Even bigger uncertainties come from the hadronic matrix elements of the tree and penguin operators due to the nonperturbative QCD effects. We have worked in the factorization approach, with the effective parameter N_c being introduced to account for the nonfactorizable effects. We have shown that the CP violating asymmetry in this decay process strongly depends on the parameter N_c .

In order to determine the range of N_c we have compared the theoretical values and the experimental data for the branching ratios for $\bar{B}^0 \rightarrow \rho^0\rho^0$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$. We have found that the latest experimental data constrain N_c to be in the range (2.74, 4.77) for $q^2/m_b^2=0.3$ and (2.81, 4.92) for $q^2/m_b^2=0.5$, respectively, when we let the CKM matrix elements vary in the ranges determined by the current experiments. We have studied the sign of $\sin\delta$ in the

range of N_c and found that $\sin\delta$ changes its sign at the point $N_c = 3.68$. This also leads to the change of the sign of the CP violating asymmetry. Due to the large errors in the current experimental data for the branching ratios for $\bar{B}^0 \rightarrow \rho^0 \rho^0$ and $\bar{B}^0 \rightarrow \rho^+ \rho^-$ we cannot constrain N_c more accurately at present. If the future experimental data could constrain N_c to be less than 3.68 (N_c is usually less than 3.68 in other studies), the sign of the CP violating asymmetry would remain unchanged in the whole range of N_c . Then one could remove the mod (π) ambiguity in the determination of the CP violating phase angle α by measuring the CP violating asymmetry in $\bar{B}^0 \rightarrow \rho^0(\omega) \rho^0(\omega) \rightarrow \pi^+ \pi^- \pi^+ \pi^-$.

For the decay process $\bar{B}^0 \rightarrow \rho^0(\omega) \rho^0(\omega)$, the factorization approach we have used is expected to be a good approximation since B meson decays are energetic and since $\alpha_s(m_b)$ and $1/m_b$ corrections should be small in the QCD factorization scheme. One may also work in the QCD factorization scheme, taking the value of N_c to be 3 and including corrections of order $\alpha_s(m_b)$ as done in Ref. [31]. However, the QCD factorization scheme suffers from endpoint singularities which are not well controlled. The CP violating asymmetry depends on the unknown parameters which are associated with such endpoint singularities. This lead to very uncertain CP violating asymmetries in the QCD factorization scheme [31]. As mentioned before, the uncertainty for the CP violating asymmetry is also very large in the factorization approach we have used, i.e. from about -96% to about 56% depending on the value of N_c and the CKM matrix elements. Furthermore, the CP violating asymmetry may strongly depend on the factorization approach adopted [31]. All these issues need further and more careful investigations.

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Figure Captions

Fig. 1 The CP violating asymmetry, a , for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. (a): for $q^2/m_b^2 = 0.3$, $N_c = 2.74$ and 4.77 and limiting values of the CKM matrix elements, ρ and η : the solid line (dotted line) corresponds to $N_c = 2.74$ and maximum (minimum) ρ and η ; the dashed line (dot-dashed line) corresponds to $N_c = 4.77$ and maximum (minimum) ρ and η . (b): for $q^2/m_b^2 = 0.5$, $N_c = 2.81$ and 4.92 and limiting values of ρ and η : the solid line (dotted line) corresponds to $N_c = 2.81$ and maximum (minimum) ρ and η ; the dashed line (dot-dashed line) corresponds to $N_c = 4.92$ and maximum (minimum) ρ and η .

Fig. 2 Comparison between the CP violating asymmetries in the cases where single and double $\rho-\omega$ mixing is involved, respectively. (a): for $q^2/m_b^2 = 0.3$ and $N_c = 2.74$, the solid line (dotted line) corresponds to the case with double $\rho-\omega$ mixing and maximum (minimum) ρ and η ; the dashed line (dot-dashed line) corresponds to the case with single $\rho-\omega$ mixing and maximum (minimum) ρ and η . (b): same as (a) but for $q^2/m_b^2 = 0.5$ and $N_c = 2.81$.

Fig. 3 $\sin\delta$ as a function of \sqrt{s} for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. (a): for $q^2/m_b^2 = 0.3$ and $N_c = 2.74(4.77)$: the solid line (dashed line) corresponds to the case with double $\rho-\omega$ mixing; the dotted line (dot-dashed line) corresponds to the case with single $\rho-\omega$ mixing. (b): same as (a) but for $q^2/m_b^2 = 0.5$ and $N_c = 2.81(4.92)$.

Fig. 4 The ratio of penguin to tree amplitudes, r , as a function of \sqrt{s} , for limiting values of ρ and η : the solid line (dotted line) corresponds to the case of double (single) $\rho-\omega$ mixing with maximum ρ and η ; the dashed line (dot-dashed line) corresponds to the case of double (single) $\rho-\omega$ mixing with minimum ρ and η . In (a) $q^2/m_b^2 = 0.3$, $N_c = 2.74$ (left) and 4.77 (right) while in (b) $q^2/m_b^2 = 0.5$, $N_c = 2.81$ (left) and 4.92 (right).

Fig. 5 The CP violating asymmetry, a , as a function of N_c and \sqrt{s} , for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ for $\rho = \rho_{max}$ and $\eta = \eta_{max}$. (a) and (b) correspond to $q^2/m_b^2 = 0.3$ and $q^2/m_b^2 = 0.5$, respectively.

Fig. 6 $\sin\delta$ as a function of \sqrt{s} and N_c . (a) and (b) correspond to $q^2/m_b^2 = 0.3$ and $q^2/m_b^2 = 0.5$, respectively.

Fig. 7 Branching ratio for $\bar{B}^0 \rightarrow \rho^0\rho^0$ for all the models when $q^2/m_b^2 = 0.3$, $\lambda = 0.2272$,

$A = 0.818$, $\rho = 0.246$, and $\eta = 0.334$: the lower (upper) solid line corresponds to Model 1 (2), the lower (upper) dotted line corresponds to Model 3 (4) and the lower (upper) dashed line corresponds to Model 5 (6).

Fig. 8 Branching ratio for $\bar{B}^0 \rightarrow \rho^+ \rho^-$ for all the models when $q^2/m_b^2 = 0.3$, $\lambda = 0.2272$, $A = 0.818$, $\rho = 0.246$, and $\eta = 0.334$: the lower (upper) solid line corresponds to Model 1 (2), the lower (upper) dotted line corresponds to Model 3 (4) and the lower (upper) dashed line corresponds to Model 5 (6).

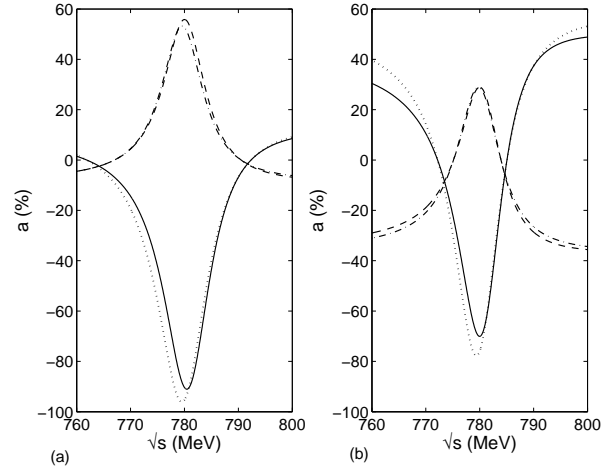


Fig. 1

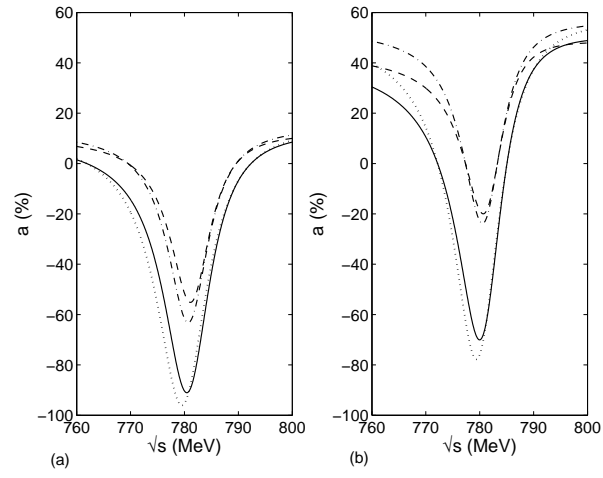


Fig. 2

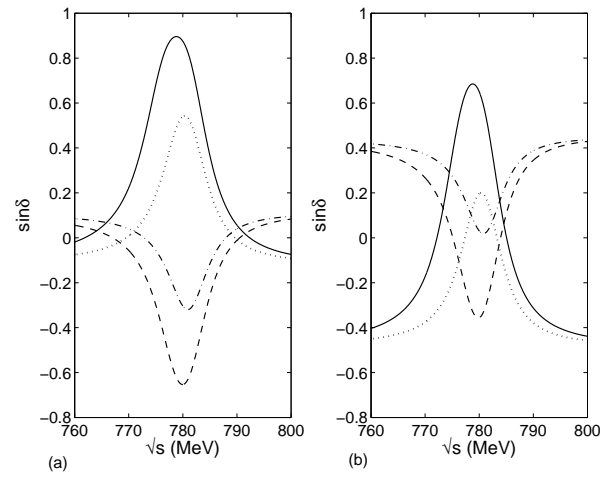


Fig. 3

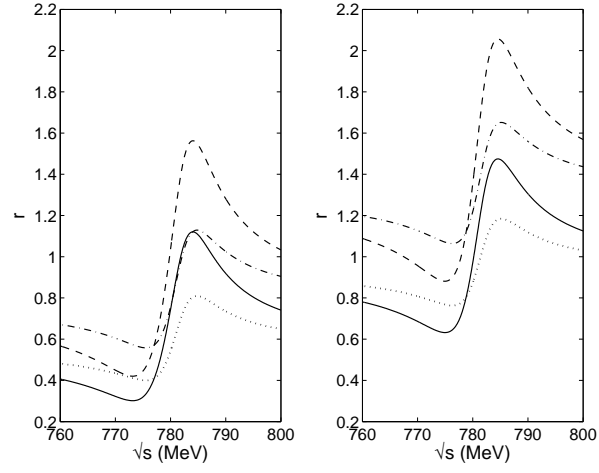


Fig. 4(a)

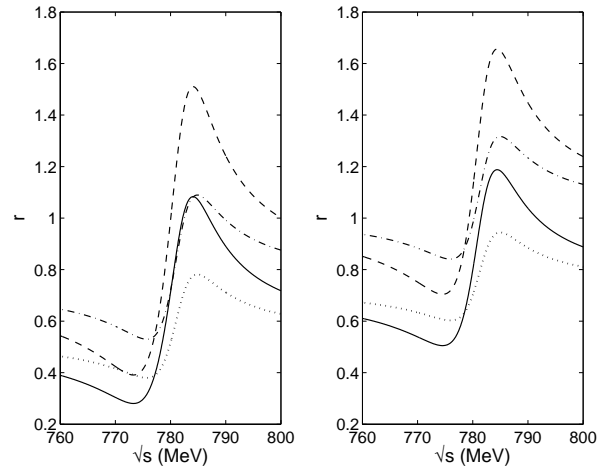


Fig. 4(b)

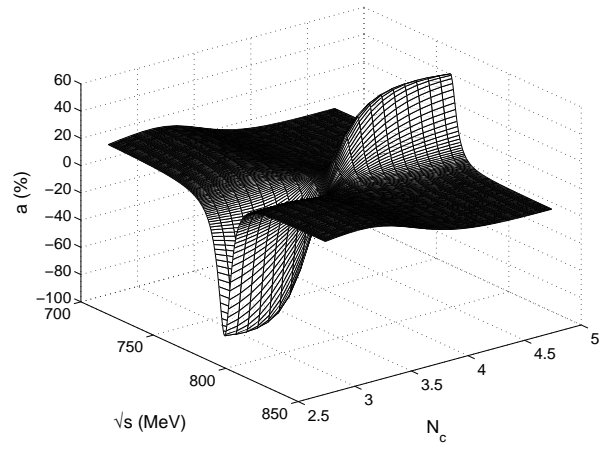


Fig. 5(a)

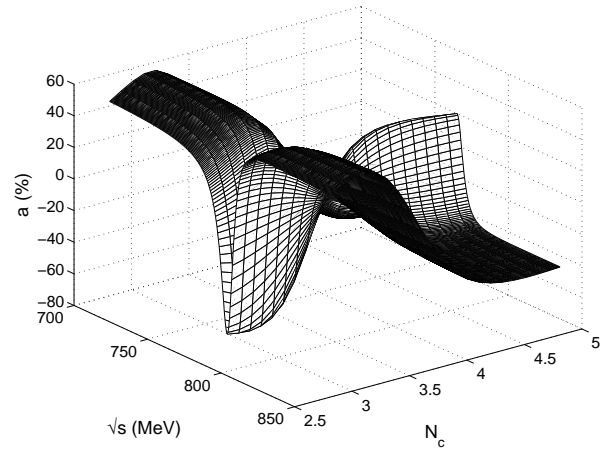


Fig. 5(b)

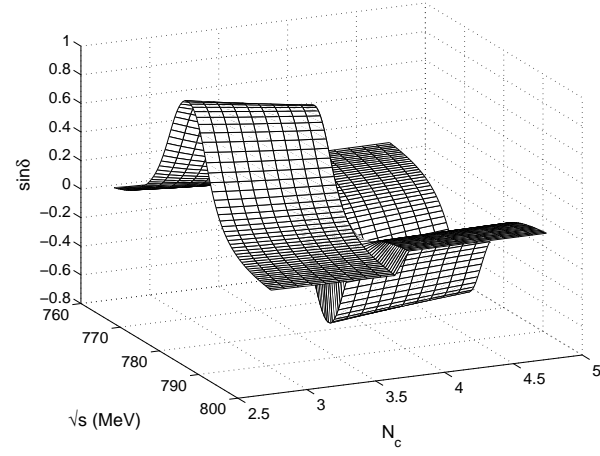


Fig. 6(a)

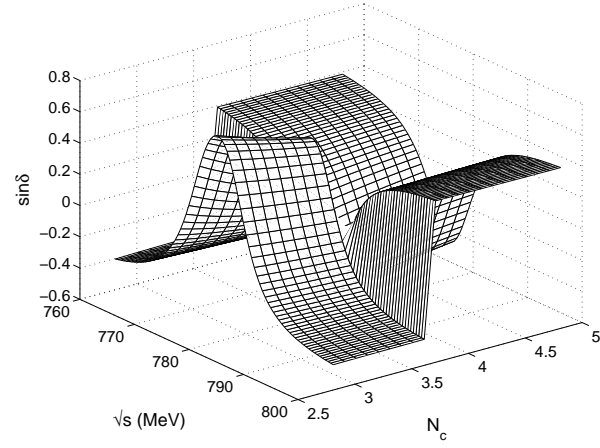


Fig. 6(b)

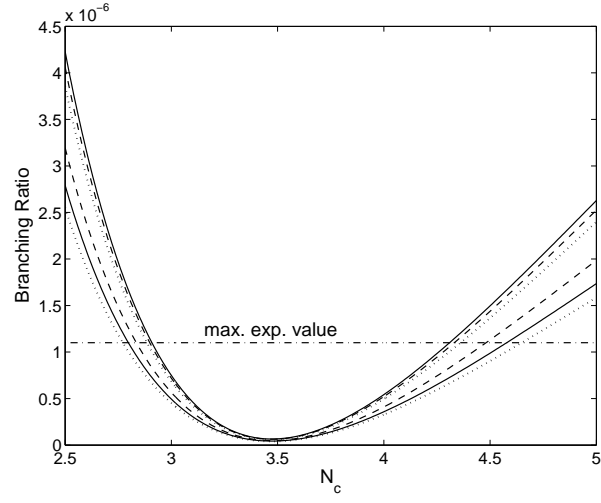


Fig. 7

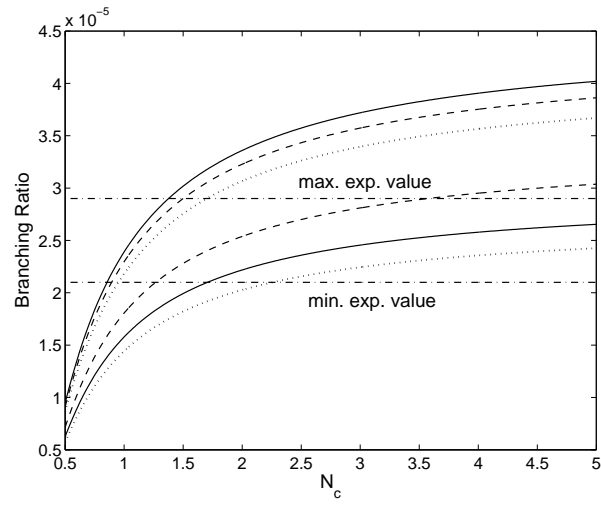


Fig. 8